Unit 2: Geometrical properties of individual features

1: Introduction

2: Individual properties (geometry)

3: Spatial pattern (relationships)

1 Introduction

Description of spatial properties (1)

Production of indices describing spatial properties

- **Individual spatial properties:**
  - Geometry: location, size, shape
- **Spatial arrangement of features (pattern):**
  - Spatial relationships: distribution, neighborhood, proximity
- **Distinction between object mode and image mode:**
  - object mode: units of observation are features (object)
  - image mode : regions are features

Description of spatial properties (2)

Described spatial features are models of those from the reality

- **Their geometry is simplified:**
  - in object mode: through the use of geometrical primitives
  - in image mode: through the choice of a resolution
- **Their attributes corresponds to a global thematic property:**
  - in object mode: point, linear, areal units
  - in image mode: the set of areal units (cells) composing the region

Description of spatial properties (3)

Descriptors summarize some properties of spatial features

- **They are indicators (indices):**
  - expressing some properties, not all
  - dependent on the quality of the modeled features
- **Such indicators should be considered as estimators of properties:**
  - several indicators can express the same property
  - their use and interpretation should be made with sound understanding

2 Individual properties of point features

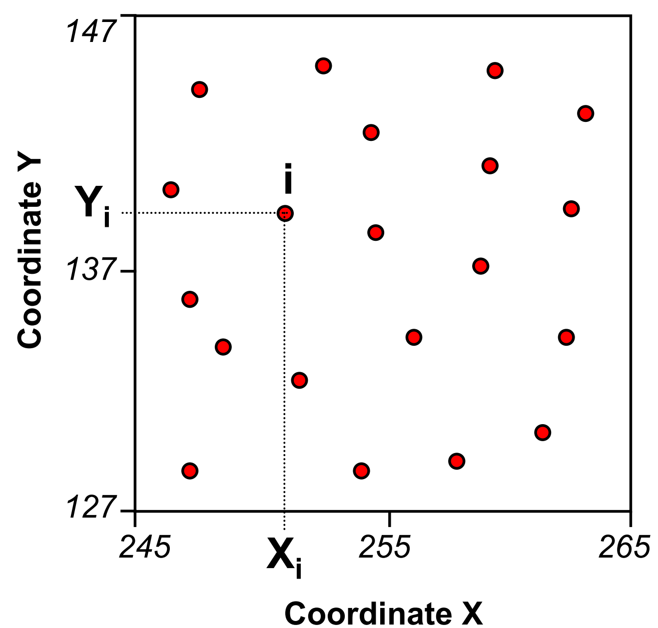
Point features: individual properties

- **A point feature is modeled as a geometrical point, it has no spatial dimension (0D)**
- **In image mode the point region is also considered as spatially dimensionless, despite the fact it is made of an areal spatial unit (the cell)**
- **Its single individual geometric property is:**
    - its location

Point object: Location (position)

- **Horizontal (X coordinate) and vertical (Y coordinate) positions in the projected plane, using a defined coordinate system**
- **<u>Example</u>:**

  Location of object i:
  (251.18m, 139.54m)



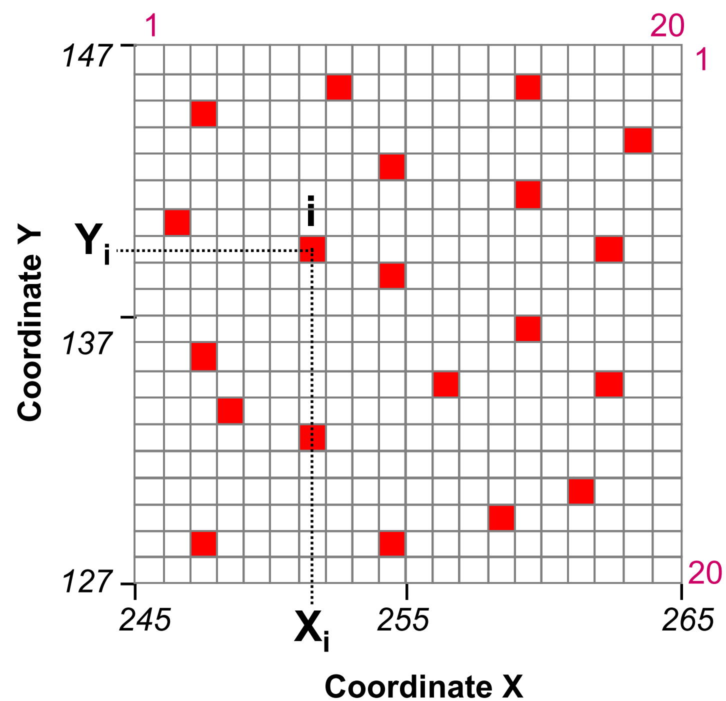
X_i = 251.18 Y_i = 139.54

Point region: Location (position)

- **Horizontal and vertical position :**
  - located at center of the cell (X,Y coordinates)
  - corresponding to the cell position in the grid (column and row)
  - grid resolution dependent
- **Example**:

  Location of region i:
  (251.5m, 139.5m), with a grid resolution of 1m

  or (7,8) in grid coordinates (column, row)



X_i = 251.5 Y_i = 139.5
ou C_i = 7 R_i = 8

3 Individual properties of linear features

Linear feature: nature and type

- **A linear feature is modeled as a geometrical broken line or a chain, it has one spatial dimension (1D)**
- **In image mode a linear region is a set of contiguous cells having only one spatial dimension too**
- **A linear feature can be:**
  - simple: made of a single chain
  - complex: made of several chains

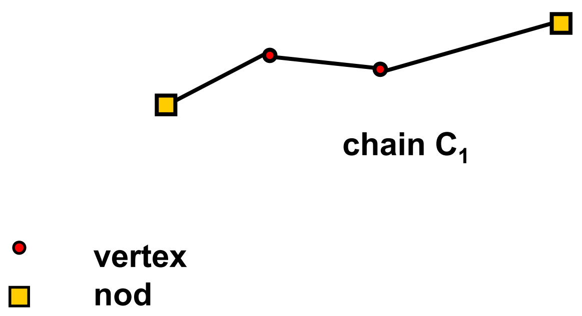
A network can be considered either as:

- a single feature (complex linear feature)
- a group of numerous features with connections

Linear feature: simple and complex

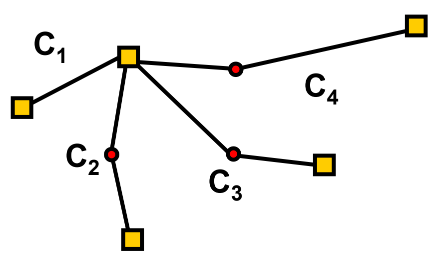
Simple linear object

- Object made of a single chain (C_1)
- Example: a segment



Complex linear object

- Object made of several chains (C_1, ..., C_n)
- Example: a set of segments, a whole network



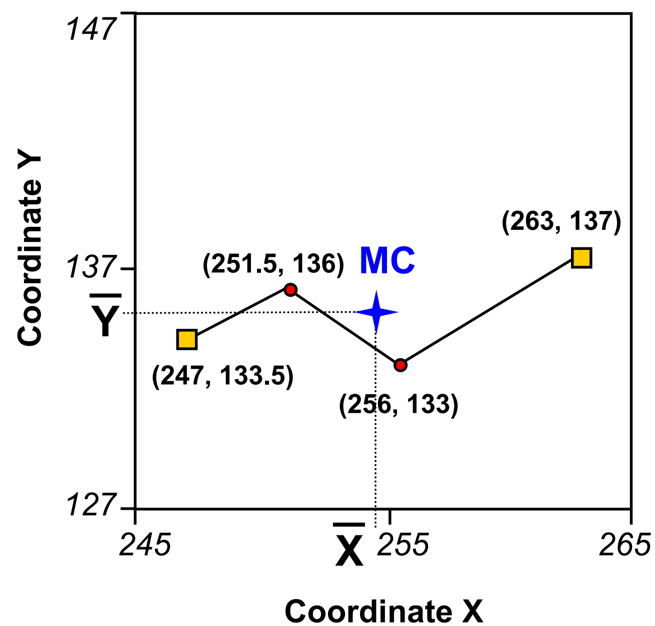
Linear feature: individual geometric properties

- **Only simple linear feature will be discussed here**
- **Set of linear features such as network will be discussed in Lesson « Network description »**
- **Individual geometric properties of a linear feature are:**
  - its location (position)
  - its size (length)
  - its shape (sinuosity)
  - its orientation (direction)

Linear object: Location

- Generally its location is considered as the horizontal and vertical position into the project plane of its Mean center (MC) or so called Gravity center
- <u>Example</u>:

Point	X	Y
1	247	133.5
2	251.5	136
3	256	133
4	263	137
Σ	*1017.5*	*539.5*
MC	254.4	134.9



$\bar{X} = 254.4 \quad \bar{Y} = 134.9$

Linear object: Size (length)

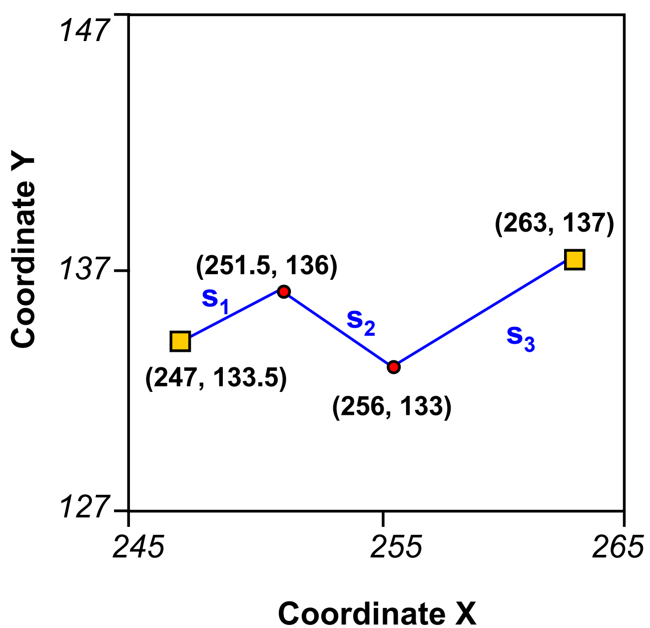
- **It is the sum of length of the n segments composing the chain:**

  $L = \Sigma Ds_i$

  with: Ds_i = distance between the two segment i ends (vertices)

- **Exemple:**

Segment	Δx	Δy	Δx^2	Δy^2	$\Delta x^2 + \Delta y^2$	Ds_i
1	4.5	2.5	20.25	6.25	26.5	5.15
2	4.5	3	20.25	9	29.25	5.41
3	7	4	49	16	65	8.06
					Σs_i =	18.62



L = 5.15+5.41+8.06 = 18.62m

Linear region: Size (length)

- **It is the sum of length of the n units (cells) composing the region:**

  This metric uses 2 yardsticks :

  diagonal = 1.41 unit, side = 1 unit

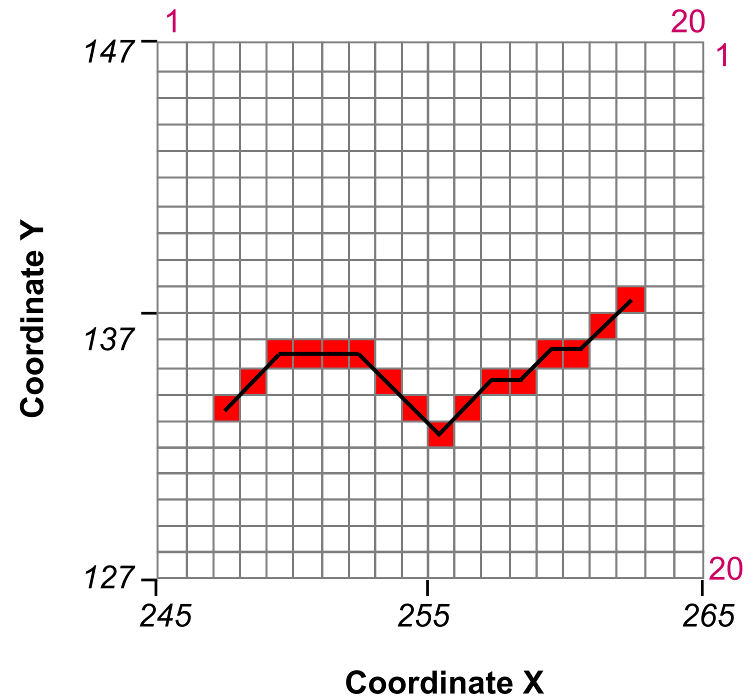
- **Example:**

  Let a grid with 1m resolution:

  L=1.41+1.41+1+1+1+1.41+1.41+1.41+1.41+1.41+1+1.41+1+1.41+1.41

  L = ((10*1.41)+(5*1))*1m = 19.1m

In image mode the estimation of length is systematically exaggerated (see the estimation in object mode: L = 18.62m)



L = ((10*1.41)+(5*1))*1m = 19.1m

Linear object: Shape (sinuosity)

- **It is the ratio between the chain length L and the distance D_{df} between its two ends:**

  $S = L / D_{df}$

- **Interpretation:**

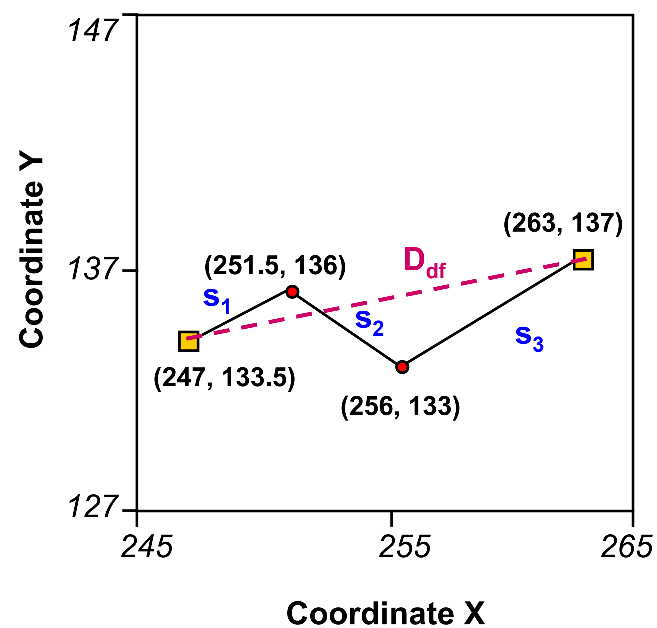
  S is a ratio, with S >=1

- **Example:**

Δx	Δy	Δx^2	Δy^2	$\Delta x^2 + \Delta y^2$	D_{df}
16	3.5	256	12.25	268.25	16.38

L = 5.15+5.41+8.06 = 18.62
D_{df} = 16.38

S = 18.62 / 16.38 = 1.14



S = 18.62 / 16.38 = 1.14

Linear region: Shape (sinuosity)

- **It is the ratio between the region length L and the distance D_{df} between its 2 ends:**

  $S = L / D_{df}$

- **Interpretation:**

  S is a ratio, with S >=1

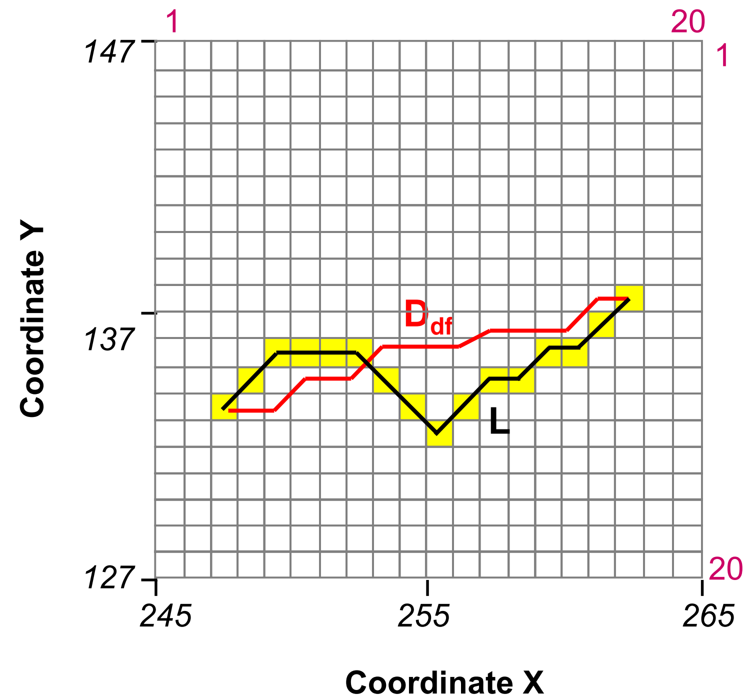
- **<u>Example</u>:**

  $L = ((10*1.41)+(5*1))*1m = 19.1m$

  $D_{df} = ((4*1.41)+(11*1))*1m = 16.64m$

  $S = 19.1 / 16.64 = 1.15$

As these 2 terms L and D_{df} use the same metric, the resulting value S is close to the one obtained in object mode (S = 1.14)



$S = 19.1 / 16.64 = 1.15$

Linear object: Orientation (direction)

- **It is the angle of the main direction of the chain with respect to the vertical**

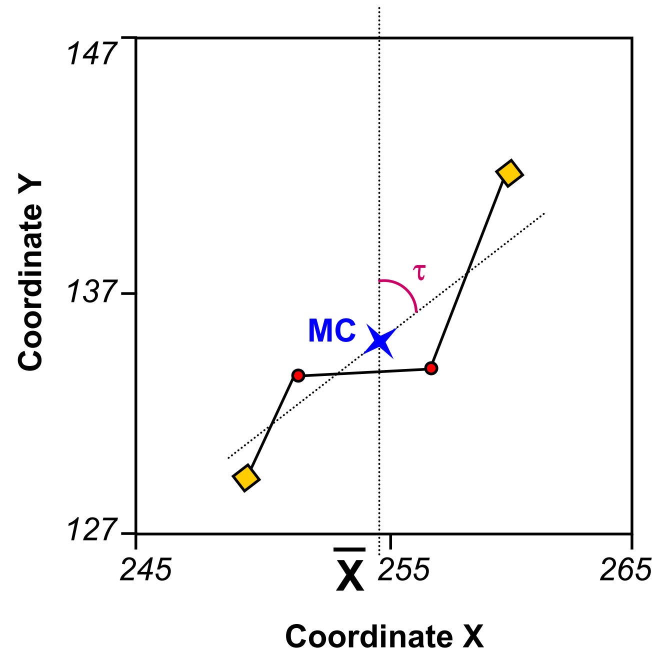
  the main direction and the vertical cross at the Mean center (MC)

- **<u>Example</u>:**

  Chain direction:

  $\tau = 56°$

  It is the same chain as in previous illustration, except its direction is different



$\tau = 56°$

4 Individual properties of areal features

Areal feature: nature and type

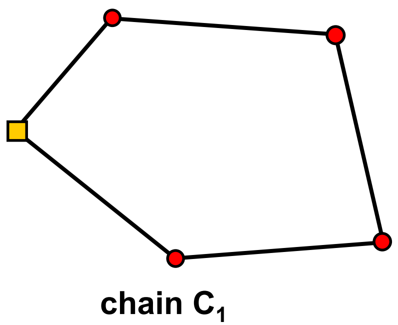
- **An areal object is modeled as a geometrical closen chain (polygon), it has 2 spatial dimensions (2D)**
- **In image mode the areal region is made of a set of contiguous cells having 2 spatial dimensions**
- **An areal feature can be:**
  - simple: made of a single polygon or region
  - complex: made of several polygons (with inner or outer area: *island*)

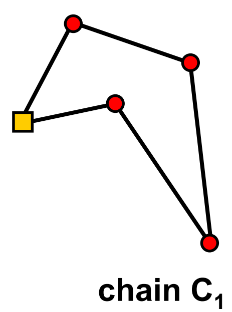
Areal object: simple and complex

Simple areal object

- Object made of a single polygon (closed chain)

Convex **Concave**

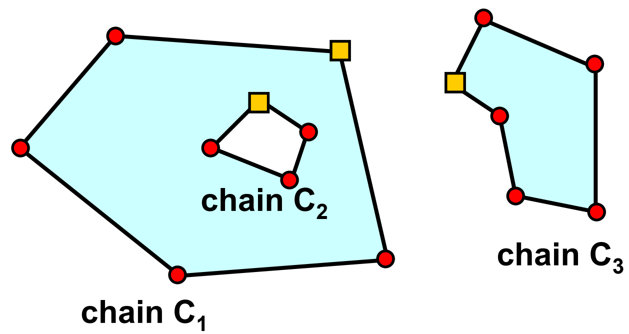




vertex
nod

Complex areal object

- Object made of several polygons (closed chains)
- Example: object with inner and outer area (islands)



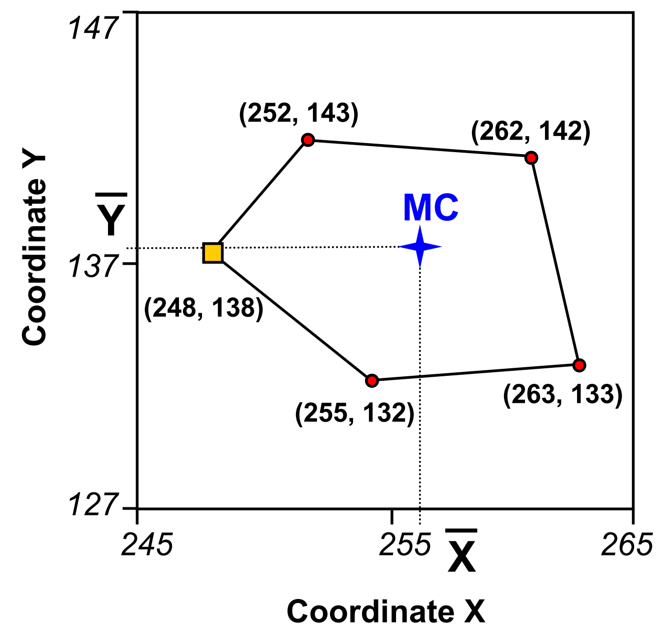
Areal feature: individual properties

- **Only simple areal features will be discussed here**
- **Individual geometric properties of an areal feature are:**
  - its location (position)
  - its size (perimeter, area)
  - its shape (compactness)

Objet zonal: Location (position)

- **Horizontal and vertical position in the projected plane of its Mean center (MC), or so called gravity center**
- **Example:**

Point	X	Y
1	248	138
2	252	143
3	262	142
4	263	133
5	255	132
Σ	*1280*	*688*
CM	256	137.6



$\bar{X} = 256 \quad \bar{Y} = 137.6$

Areal object: Size (perimeter)

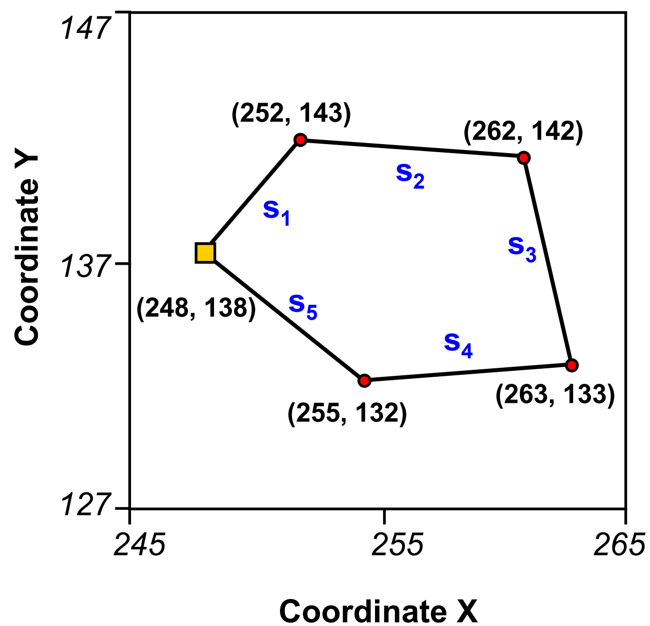
- **It is the sum of length of the n segments composing the chain:**

  $L = \Sigma Ds_i$

  with: Ds_i = distance between the two segment i ends (vertices)

- **Example:**

Segment	Δx	Δy	Δx^2	Δy^2	$\Delta x^2 + \Delta y^2$	Ds_i
1	4	5	16	25	41	6.4
2	10	1	100	1	101	10
3	1	9	1	81	82	9
4	8	1	64	1	65	8.1
5	7	6	49	36	85	9.2
					$\Sigma s_i =$	42.7



L = 6.4+10+9+8.1+9.2 = 42.7m

Areal region: Size (perimeter)

The perimeter can be evaluated with 2 different techniques:

- **External perimeter of the region (envelop) :**
  - staircase effect using "Manhattan distance"
  - systematic over estimation of the perimeter
- **Length of the linear region edge (linear perimeter) :**
  - reduced staircase effect by taking the diagonal distance into account
  - under estimation of the perimeter with a too coarse resolution

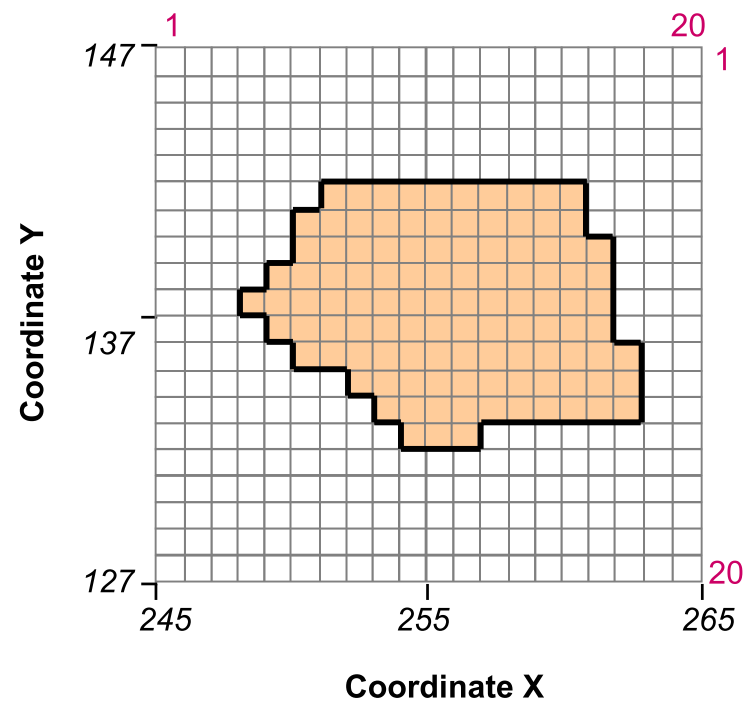
These two estimation are dependant on the metric used and the grid resolution

Areal region: Size (perimeter of the envelop)

- **It is the sum of length of the n cell sides bounding the region:**
  This metric distance is often called Manhattan
  yardstick: side = 1 unit
- **Example:**
  Let a grid with 1m resolution (unit):

  L = (50*1)*1m = 50m

In image mode, the use of this metric tends to over estimate the measure of perimeter (see estimation in object mode: L = 42.7m)



L = (50*1)*1m = 50m

Areal region: Size (linear perimeter)

- **It is the sum of length of the n cells bounding the region :**

  This metric uses 2 yardsticks:

  diagonal = 1.41 unit, side = 1 unit

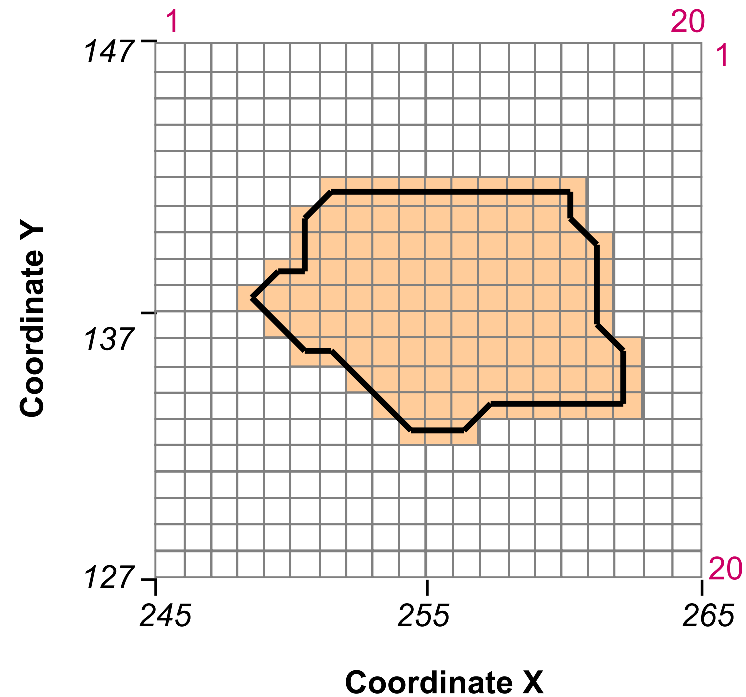
- **Example:**

  Let a grid with 1m resolution (unit):

  L=1.41+1.41+1+1.41+1+1+1+1+1+1+1+
  1+1+1+1+1.41+1+1+1+1.41+1+1+1+
  1+1+1+1+1+1.41+1+1+1.41+1.41+
  1.41+1+1.41+1.41

  L = ((11*1.41)+(24*1))*1m = 39.51m

The estimation of perimeter using this technique is close to the one in object mode (L = 42.7m)



L = ((11*1.41)+(24*1))*1m = 39.51m

Areal object: Size (area)

There are several techniques to estimate the area of an object:

- **For non generalized features (eg. features manually delineated on a map or an image):**
  - random or regular point sampling technique (Unwin D., 1981, p.126)
  - assuming objects are already generalized into polygons, this technique will not be discussed here
- **For areal objects numerically described with polygons (for a GDB in object mode):**
  - by breaking up into triangles
  - by breaking up into rectangles (computer algorithm)

Areal object: Size (area) - split into triangles

- **The area of polygon A_p is the sum of areas of its composing triangles A_{ti} :**

  $$A_p = \Sigma A_{ti}$$

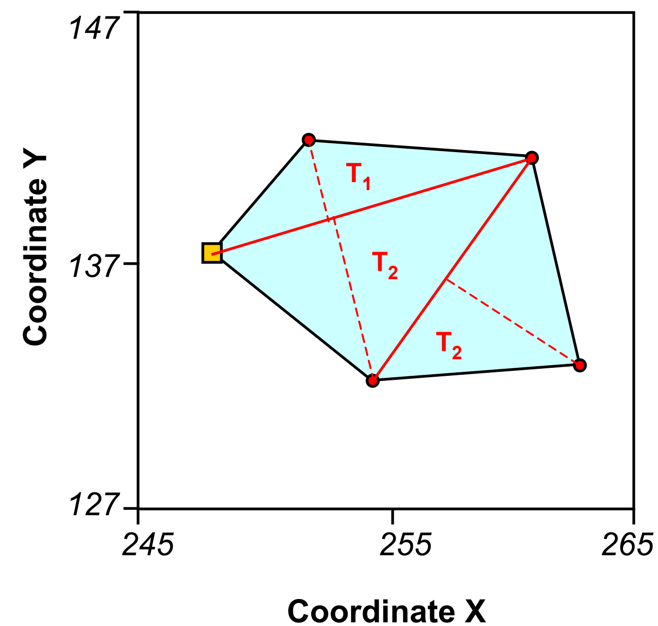
  with: $A_{ti} = (B*h) / 2$,

  B the basis and h the height

- **Example:**

Triangle	Basis	Height	B x h	$Area_{ti}$
1	14.5	3.7	53.65	26.82
2	14.5	7.7	111.65	55.82
3	12.3	6	73.8	36.9
			ΣA_{ti} =	119.54

A_p = 26.82+55.82+36.9 = 119.54m²



A_p = 26.82+55.82+36.9 = 119.54m²

Areal object: Size (area) - split into rectangles

This technique is adapted for a numerical vector structure (GDB)

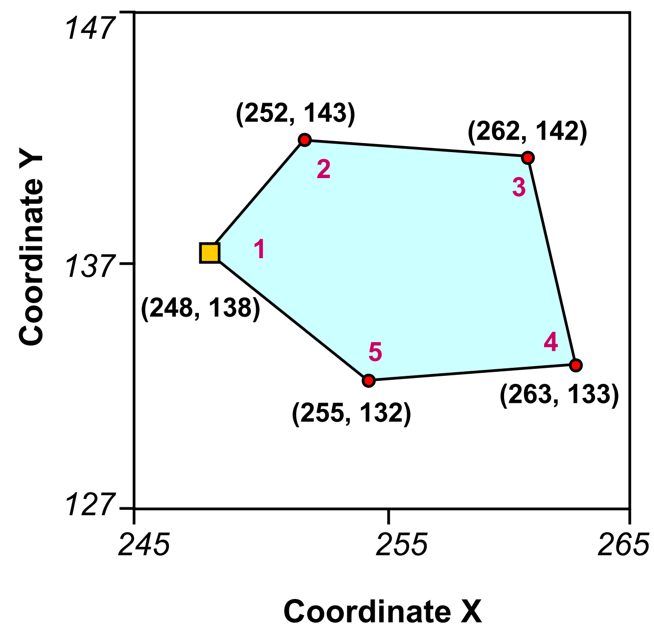
Assuming polygon vertices are sequentially described clockwise from a given starting point:

$$A = 0.5 * \Sigma y_i (x_{i+1} - x_{i-1})$$

- **Example:**

Xi	Y_i	Contribution
248	138	138(252-255) = -414
252	143	143(262-248) = 2002
262	142	142(263-252) = 1562
263	133	133(255-262) = -931
255	132	132(248-263) = -1980
	Σ contributions =	239

A = 0.5 * 239 = 119.5m²



A = 0.5 * 239 = 119.5m²

Areal region: Size (area)

- **It is the sum of areas of cells composing the region:**

  $A = n * A_u$

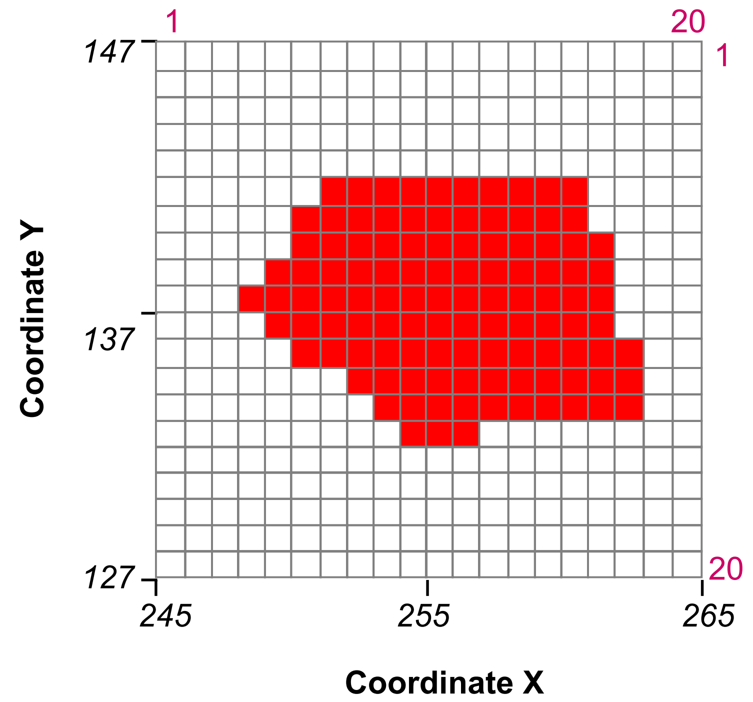
  with A_u constant for the n units

- **<u>Example</u>:**

  Let a grid with $1m^2$ resolution (unité):

  $A = 110 * 1m^2 = 110m^2$

In image mode, estimation of the the area is close to the one in object mode: ($A = 119.5m^2$)



$A = 110 * 1m^2 = 110m^2$

Areal feature: Shape (indices)

The shape of areal features is a very rich concept that is difficult to summarize with a single index

- **Such indices should allow the comparison between features:**
  - independant of the description scale and the size of features
  - with a reference to a particular shape

➜ **This index should be a ratio with at least one reference value**

Areal feature: Shape (compactness indices)

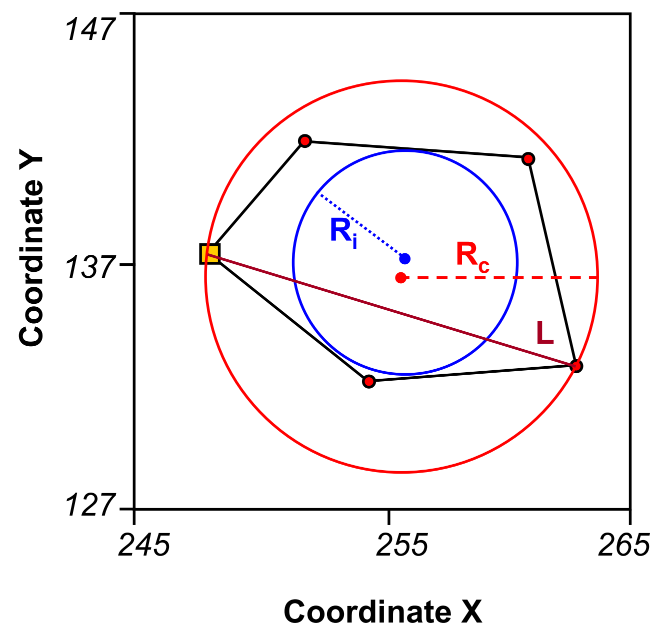
Among the numerous indices proposed in the literature, those describing the compactness of the shape

- ***Counter-example:* perimeter / area index (P/A)**
  - it is simple to produce (based on size indices)
  - but its value is dependant on the unit of measurement as well as on the size of features. its use is therefore strongly limited for the comparison of features compactness

➔ **A compactness index refers to a geometrically compact shape, such as a circle or sometimes a square**

Basic elements for compactness indices

- **For the concerned feature:**
  - A : area of the feature
  - L : major axis (distance between the 2 most faraway vertices of the feature)
- **For the reference feature (circle):**
  - C : area of smallest circumscribing circle
  - R_C: radius of smallest circumscribing circle
  - I : area of largest inscribed circle
  - R_I: radius of largest inscribed circle
- **Example:**
  - $A = 119.5m^2$ $L = D_{1,4} = 15.8m$
  - $R_C = 8m$ $C = \pi R^2 = 201.06m^2$
  - $R_I = 5.1m$ $I = \pi R^2 = 81.71m^2$



R_c : radius of circumscribing circle C
R_i : radius of inscribed circle I
L : major axis of feature

Areal feature: Compactness indices

Most usual compactness indices are made of:

- **The ratio between the feature area and the area of its smallest circumscribing circle :**
  - $S_{A,C} = A / C$
- **The ratio between the feature area and the area of a circle having the major axis length L as perimeter :**
  - $S_{A,L} = A / \pi (0.5 L)^2 = 1.27 A / L^2$
- **The ratio between the largest inscribed circle area and the area of its smallest circumscribing circle :**
  - $S_{I,C} = I / C$

Areal feature: Compactness indices (continued)

Some other indices derived :

- **From the ratio between the feature area and the area of its smallest circumscribing circle :**
  - $S'_{A,C} = \sqrt{(A / C)}$ $Sr_{A,C} = R_A / R_C$, with $R_A = \sqrt{(A / \pi)}$
- **From the ratio between the largest inscribed circle area and the area of its smallest circumscribing circle :**
  - $Sr_{I,C} = R_I / R_C$
- **From the ratio between the minor and the major axis:**
  - $S_{l,L} = l / L$

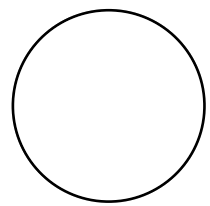
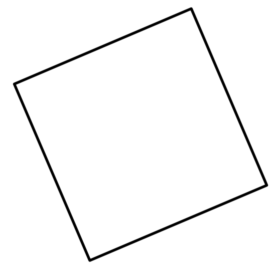
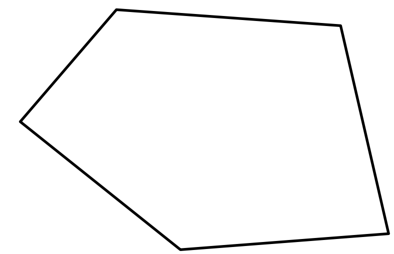
    with l being the minor axis, perpendicular to the major axis

Areal feature: Compactness indices (continued)

And some easily computable indices

- **Some basic elements involved in the computation of compactness are difficult or tedious to produce for irregular features:**
  - particularly inscribed and circumscribing radius
- **In numerous GIS software proposed compactness indices are therefore computed as follow:**
  - $S_{A,Cp} = A / Cp$, with Cp as the area of a circle having the same perimeter as the feature
  - $S_{A,Q} = A / Q$, with Q as the area of a circumscribing square with a side length equal to L

Areal feature: Compactness indices - Comparison

Indices	Formula	Circle	Oriented square	Irregular polygon
				
$\mathbf{S_{A,C}}$	= A/C	= 1	= 0.64	= 119.5 / 201.06 **= 0.59**
$\mathbf{S_{A,L}}$	= 1.27 A/L^2	= 1	= 0.64	= 119.5 / 196.07 **= 0.61**
$\mathbf{S_{I,C}}$	= I/C	= 1	= 0.5	= 81.7 / 201.6 **= 0.41**
$\mathbf{S'_{A,C}}$	= $\sqrt{(A/C)}$	= 1	= 0.8	= $(119.5 / 201.06)^{0.5}$ **= 0.77**
$\mathbf{Sr_{A,C}}$	= R_i / R_c	= 1	= 0.71	= 5.1 / 8 **= 0.64**
$\mathbf{S_{I,L}}$	= I/L	= 1	= 1	= 10.9 / 15.8 **= 0.69**
$\mathbf{S_{A,Cp}}$	= A/C_p	= 1	= 0.71	= 119.5 / 144.3 **= 0.83**
$\mathbf{S_{A,Q}}$	= A/Q	= 0.78	= 0.5	= 119.5 / 15.8^2 **= 0.48**

Characteristics of the irregular polygon (illustration of the areal object):
A = 119.5, L = 15.8, I = 10.9, P = 42.7, C= 201.06, R_C = 8, R_I = 5.1

Areal feature: Compactness indices - Comments

All these indices express the relative compactness of a feature with respect to a compact shape of reference

- **For all except the last index, the reference is a circular shape :**
  - the maximum value 1 expresses a maximal compactness
  - the lesser the compactness of the polygon, the lower the index value
- **Each index expresses differently the discrepancy between the feature shape and the reference shape**
  - It is therefore important to master the meaning of index values

Areal feature: Compactness indices - References

Suggested references

Baker L., :

Davis P., :

Ebdon D., :

Fitzgerald B., :

Hammond, Mc Cullagh, :

Unwin D., :

Idrisi (Cratio) :

ArcGIS :

5 Arrangement spatial des objets ponctuels

Fin de l‘ Unité